

# Using a Kinematic Definition of the Hubble Parameter to Determine the Cosmological Constant $\Lambda = 0$ in a Balanced Universe

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The Hubble parameter is kinematically defined in terms of the positions and velocities of all particles in a universe which may or may not be finite. This definition is set equal to the Hubble parameter as defined in the Friedman-Lemaître solution of general relativity, and which occurs after the inflationary expansion has ended in the Guth model. Because a coordinate system at rest relative to its local Hubble drift is a system in which the cosmic background radiation is observed to be isotropic, it is also an inertial system. Just before the first mass particles are created within a pure radiation universe, there are no mass particles that exist which can define  $H$  or the inertial systems associated with the Hubble drift. It will be shown that only a cosmological constant with a magnitude of zero will allow radiation to form mass particles that have a total energy which is independent of inertial systems and is equal to the equivalent energy of their rest mass. Additional mass particles are continuously formed from the radiation throughout the expanding universe after the initial particles are created.

## I. Introduction

The Hubble parameter  $H$  and the recessional motion in an expanding universe describes the receding velocity  $v$  between any two galaxies and the distance between them  $r$  to be related to each other by the relation  $v = Hr$ . This occurs exactly when neither galaxy has a peculiar velocity which would have been caused by a local force such as gravitation. Current observations indicate that this expansion of the universe appears to be either balanced with the velocity of the receding galaxies either completely vanishing or approaching a very small magnitude as the distance between all the galaxies approach infinity. This is a peculiar occurrence because it would indicate that the initial recessional velocities of the galaxies would be required to have a magnitude that provides a kinetic energy that is exactly enough to allow it to overcome gravity and expand while coming to rest at infinity with extremely little or no residual velocity. No physical effect is known that could cause such a result.

The purpose of this paper will be to show that when inertial systems are defined to be systems which are either at rest or moving with a constant velocity relative to their local Hubble drift, they are kinematically defined by the Hubble parameter. The standard relativistic expansion occurring after the inflationary expansion in the Guth model [1] requires that  $\Lambda = 0$  so that the expansion is exactly balanced. It will be shown that this is required to occur when the Hubble parameter itself can be kinematically defined as an explicit function of the positions and velocities of all mass particles in the universe.

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The Hubble parameter can only be defined in terms of mass particles and cannot be defined in terms of photons for the following reason: Although radiation density in general relativity causes gravitational decelerations and can change the magnitude of  $H$ , it is only mass particles whose positions and velocities alone can be used in a kinematic definition of the Hubble parameter and not the positions or velocities of the radiation's photons because they have a fundamentally different behavior. A mass particle's velocity refers to the motion of its rest mass, and its motion relative to other mass particles can be measured by the change in interparticle distances between masses or relative to an observer, and does not require reference to inertial systems. The velocity  $c$  of a photon has physical meaning only in reference to the local inertial system in which it is moving, and observers at rest in different inertial systems which are moving relative to each other see different frequencies and energies for a photon which could be used to define the equivalent rest mass of the photon. The definition of a distant photon's velocity  $v$  cannot be defined in terms of the rate of change of interparticle distances alone, but only in terms of its peculiar velocity  $c$  relative to the receding inertial system in which it is located and which is then added to the recessional Hubble velocity  $Hr$  of the system itself so that an approaching photon's velocity is  $v = Hr - c$ . The photon's velocity depends on  $H$  and therefore cannot be used in turn within the definition of the magnitude of  $H$  which determines the inertial system's receding velocity. It is for this reason that the kinematic expression of  $H$  explicitly depends only upon the locations and motion of mass particles.

As with photons, the peculiar velocity of mass particles also cannot be used in the definition of  $H$ . The peculiar velocity of a particle is defined as its velocity relative to the local Hubble drift in which it is located. It therefore depends on the Hubble drift and this requires, of course, that  $H$  be defined before-hand. If the definition of  $H$  is to be independent of abstract concepts and depend only on observable physical quantities that exist in the universe, the sole assumption on which the cosmological kinematics of the Hubble parameter can be based is as follows: *An equation defining the magnitude of  $H$  and describing its kinematic behavior must only be a function of the physical vector lengths that define the positions of all existing mass particles in the universe, and the velocities associated with these lengths.* The definition of the Hubble parameter  $H$  will be directly determined here by explicit quantities expressing the vector position  $\mathbf{S}_k$  of each particle  $k$ , rather than a general vector function  $\mathbf{S}$  that refers to a position in space whether or not it defines the position of a particle. The lengths used will be physically measurable lengths, which are the integrals of the space components of the geodesic metric length  $ds$  of general relativity existing within what will be assumed is a universe generally consisting of flat space. The physical lengths are generally not coordinate lengths and are independent of abstract concepts used in coordinate systems. Curvature of space or mass distributions in a flat universe can distort local space-time and the lengths they contain; these distorted lengths are defined as physical lengths as explained, for example, by Sokolnikoff [2]. Local gravitational distortions of length are extremely small compared to the vast cosmological distances in the universe so

that they differ very slightly from coordinate lengths, and physical lengths are essentially used here for their conceptual importance.

## II. Kinematic and Gravitational Definitions of $H$

A general definition of the Hubble parameter can now be found by assuming it is a function of only the distances and velocities of all mass particles that at the time of the origin of the universe are relatively few in number, have all been created within a small finite volume of space by the radiation field which may be, but is not necessarily, finite and which homogeneously fills the universe. Only after the universe's initial expansion has occurred would a large and continuous creation of particles take place throughout the radiation universe and consequently produce a homogeneous distribution of matter. Photon particles are not included in the definition of  $H$  for the reason explained in the introduction. A complete kinematic description of the Hubble parameter was described by the author in a previous paper [3]. Because the cosmological distances between mass particles are large compared their size, the universe will be considered to macroscopically consist of a number  $N$  of point-like mass particles of equal rest mass  $m$ . When  $N$  is large, any two unequal macroscopic masses could simply be considered to be composed of different numbers of point-like masses. The rest mass and not the relativistic mass is used because relativistic mass is a function of its velocity relative to inertial systems which are to be defined here. The vector positions  $\mathbf{S}_k$  as used here will always represent the distance of a particle  $k$  having a rest mass  $m$  within a spatially flat universe which may contain intermittent local volumes of curved space-time due to local gravitational distortions generated by stars or galaxies. It represents the spatial three-dimensional metric length along a geodesic path that is the physical length running from the center of the expansion, which is the center of the initial mass distribution, to the position of particle  $k$ . Assuming the expansion initially begins within a very small volume of space containing all the mass particles within an otherwise radiation universe, a continuing process of particle creation throughout the radiation universe would subsequently occur and can be shown to quickly reduce the physical effects of the initial particles located at the center of the expansion. According to the superposition of the two kinds of motion that occur on the cosmological level, the velocity of a particle is then defined as

$$\dot{\mathbf{S}}_k = H\mathbf{S}_k + \dot{\mathbf{s}}_k. \quad (1)$$

Here,  $\dot{\mathbf{s}}_k$  is the peculiar velocity of particle  $k$  which would be observed as the particle's velocity relative to the local inertial system that is itself at rest relative to the radial Hubble velocity  $H\mathbf{S}_k$  at the particle's location. The unit vector along the direction of  $\mathbf{S}_k$  is defined as  $\mathbf{k} = \mathbf{S}_k/S_k$ . The same letter  $k$  is used here to represent both the unit vector  $\mathbf{k}$  as well as the subscript in the letter defining the particle's position, however they are easily distinguished because one is a vector and the other a scalar.

The time  $t$  contained in the velocities is here defined as the cosmic time measured by a clock that is at rest relative to the Hubble drift in which the mass

particle is located. As assumed in relativity theory and the cosmological principle, the laws of physics are the same everywhere. It must then be expected that time  $t$  measured by a clock at rest in a galaxy would flow at the same rate in every galaxy that is at rest relative to its local Hubble drift. The Lorentz transformation and relativity then describe time transformations among coordinate systems within a single galaxy, but not between two galaxies having relative velocities generated by the Hubble expansion and containing identical clocks. It is for these reasons that the kinematic equations developed here are different from, and cannot have, the covariant form of relativity theory.

As stated in the introduction, an expression for  $H$  can be obtained which is a function of only  $\mathbf{S}_k$  and its derivative with respect to time  $\dot{\mathbf{S}}_k$ . By taking the scalar product of both sides of Eq. (1) with  $\mathbf{k}$  and summing over all values of  $k$  from 1 to  $N$ , an expression for the Hubble parameter in the form

$$H = (\Sigma \dot{\mathbf{S}}_k \cdot \mathbf{k}) / (\Sigma \mathbf{S}_k \cdot \mathbf{k}) \quad (2a)$$

is found when the important condition

$$\Sigma \dot{\mathbf{S}}_k \cdot \mathbf{k} = 0 \quad (2b)$$

is satisfied by all the radial components of the peculiar velocities  $\dot{\mathbf{S}}_k$ . The terms in Eqs. (2a) and (2b) are scalar functions which represent only the radial components of  $\mathbf{S}_k$ ,  $\dot{\mathbf{S}}_k$ , and  $\dot{\mathbf{S}}_k$  due to the scalar product with  $\mathbf{k}$ . Once the magnitude of  $H$  is defined by Eq. (2a), the magnitudes of the peculiar velocities  $\dot{\mathbf{S}}_k$  are also determined by Eq. (1) in terms of  $\dot{\mathbf{S}}_k$  and  $H\mathbf{S}_k$ . The definition of  $H$  in Eq. (2a) cannot contain the peculiar velocities  $\dot{\mathbf{S}}_k$  for the same reason that it cannot contain references to photon velocities because both are defined relative to their inertial system's receding Hubble motion. The definition of the expansion parameter  $a(t)$  is given by  $\mathbf{S}_k = \mathbf{h}_k a$ , where  $\mathbf{h}_k$  is a vector length that is independent of  $a$  and is associated with particle  $k$  as  $\mathbf{S}_k$  expands as a linear function of  $a$ . When it is substituted into Eq. (2a) it reduces to the general definition of the Hubble parameter  $H = \dot{a}/a$  by recognizing that the peculiar velocity is  $\dot{\mathbf{S}}_k = \dot{\mathbf{h}}_k a$  which in turn must satisfy Eq. (2b). Consequently Eqs. (2a) and (2b) are then justified as a physically correct definition of  $H$ . Of course, if  $\dot{\mathbf{S}}_p = H\mathbf{S}_p$  and  $\dot{\mathbf{S}}_q = H\mathbf{S}_q$  are the Hubble velocities for particles  $p$  and  $q$  receding relative to the center of the expansion, then the difference between these two equations  $\dot{\mathbf{S}}_p - \dot{\mathbf{S}}_q = H(\mathbf{S}_p - \mathbf{S}_q)$  shows that the Hubble recession also occurs between any two particles that have no peculiar velocities, and does not occur only relative to the center of the expansion.

If particles are joined together to form particles of different masses, these equations then contain weighted positions and velocities. When the numerator and denominator in Eq. (2a) are each divided by  $N$ , it is seen that the equation represents the average radial speed of the mass particles divided by their average distance from the center of the expansion. When Eq. (2b) is multiplied by the mass  $m$  it has a form somewhat similar to the conservation of momentum arising from Newton's third law of motion concerning action and reaction. However, it is not a Newtonian equation because each particle located at  $\mathbf{S}_k$  has

a different direction for its unit vector  $\mathbf{k}$ . It then requires that the sum of only radial momentums of the peculiar velocities vanish. This means that the local radial interaction between particles cannot change the total radial momentum of peculiar velocities or affect the magnitude of  $H$ .

The magnitude of  $H$  is shown in [3] to be kinematically changed only by the peculiar velocities  $\dot{\mathbf{s}}_k$ . Of course  $H$  is also affected by gravitation on a cosmological scale as described by general relativity. Newtonian gravitational mechanics can yield the same exact cosmological equations as found in general relativity, as was originally pointed out by Milne [4] and Milne and McCrea [5]. The gravitational equations for  $H$  are found to have the general form

$$H^2 = \frac{8\pi G}{3}\rho - \frac{c^2}{R_0^2 a^2} + \frac{\Lambda}{3}, \quad (3)$$

where  $\rho$  is the density of mass and the equivalent mass of radiation energy,  $R_0$  is a scaling constant within the curvature term,  $\Lambda$  is Einstein's cosmological constant, and  $G$  is the gravitational constant. By taking the time derivative of Eq. (3) and using Lemaître's equation

$$\dot{\rho} = -3H(\rho + p/c^2), \quad (4)$$

where  $p$  is radiation pressure, the gravitational change in the Hubble parameter is seen to be

$$\dot{H} = -4\pi G \left( \rho + \frac{p}{c^2} \right) + \frac{c^2}{R_0^2 a^2}. \quad (5)$$

Equations (2a) and (2b) define the Hubble parameter in kinematic terms involving only particle positions and velocities. Solutions to Eq. (5), with the help of Eq. (4), also define the Hubble parameter in terms of gravitation. Because the Hubble parameter is defined as  $H = \dot{a}/a$  in both cases, these two expressions for  $H$  must have identical magnitudes. This physical identity will be used to show that  $\Lambda = 0$  is a required condition in the standard Friedman-Lemaître solution of the cosmological equations in general relativity.

### III. PHYSICAL CONDITIONS GENERATING A BALANCED EXPANSION WITH $\Lambda = 0$

The Guth inflationary model involves a universe consisting of massless particles and contains an initial inflationary expansion followed by a second expansion of the kind that occurs in the Friedman-Lemaître solution. This could occur when the mass particles generating the inflationary expansion have a very short life-time so that it ends as a pure radiation model containing no mass particles. At that instant the universe could transition into one with a set of newly created mass particles that are different from the inflationary particles that had been created in a much more dense universe than those which were created after the end of the inflationary period, all of which are described here in terms of primed letters. For simplicity the particles will be assumed in this case to form

a single small spherical shell of radius  $S_k = r'_0$  on which they may or may not be uniformly distributed at the time of their creation. However, each particle is created with an outwards moving initial radial velocity  $v'_0$ , with no tangential velocities. When all the existing mass particles in the universe have the same positive radial velocity  $\dot{S}_k = v'_0$ , they would then not have peculiar velocities because Eq. (2a) then defines  $H = v'_0/r'_0$  where  $\dot{\mathbf{s}}_k = 0$  for all particles, which satisfies Eq. (2b). In the absence of peculiar velocities no radiation pressure caused by Doppler effects can act on the particles. When the energy density  $u = \rho c^2$  and the radiation pressure  $p = u/3$  are substituted into Eq. (4) it is seen that the equation is satisfied when the equivalent mass density of the homogeneous and isotropic radiation is  $\rho = \rho_0(a_0/a)^4$ , where  $\rho_0$  is the density when  $a = a_0$ .

When these expressions for  $\rho$  and  $u$  are substituted into Eq.(5) for the case of a flat universe with  $R_0 = \infty$ , and  $a'$  is used to define the post-inflationary expansion parameter, the result is

$$\dot{H} = -\kappa'_3(a'_0/a')^4, \quad (6)$$

where  $\kappa'_3 = (16/3)\pi G\rho'_0$  with a numerical value  $\kappa'_3 = 1.12 \times 10^{54} \text{s}^{-2}$  when it is assumed to contain the post-inflation equivalent mass density of  $\rho'_0 = 10^{60} \text{g/cm}^3$ . A solution to this equation occurring in the standard model of relativity, where  $H = \dot{a}/a$ , is

$$a' = (\sqrt{2\kappa'_3} t' + 1)^{1/2} a'_0, \quad (7)$$

with  $t' = 0$  occurring at the instant that the expansion of the standard model began. At the time that this second expansion begins the expansion parameter is  $a'_0 = 10^{30} a_0$ , where  $a = a_0$  at the time when inflation began.

At the end of the inflationary expansion, when the second (relativistic) expansion begins, the time derivative of Eq. (7), defines a new value for  $\dot{a}$  which is much smaller than its previous value that occurred at the end of the inflationary expansion. Substituting Eq. (7) into the right-hand side of Eq. (6) yields an equation which has the solution

$$H = \sqrt{\kappa'_3/2} \left( \sqrt{2\kappa'_3} t' + 1 \right)^{-1} + \sqrt{\frac{\Lambda}{3}} \quad (8)$$

where  $\Lambda$  is a cosmological constant. Equation (8) describes  $H$  as a function of time that originated from the use of Eq. (5), while Eq. (2a) describes it as a function of the positions and velocities of the mass particles. Because they both define the Hubble parameter as  $H = \dot{a}/a$  both of these equations can be set equal to each other so that

$$\frac{\Sigma \dot{\mathbf{S}}'_k \cdot \mathbf{k}}{\Sigma \mathbf{S}'_k \cdot \mathbf{k}} = \sqrt{\kappa'_3/2} \left( \sqrt{2\kappa'_3} t' + 1 \right)^{-1} + \sqrt{\frac{\Lambda}{3}}. \quad (9)$$

At the time  $t' = 0$  the identical radii and radial velocities  $S_k = r'_0$  and  $\dot{S}_k = v'_0$  are used for all particles on the spherical shell. The parameter  $H$  on the left-

hand side of Eq. (9) then becomes simply  $v'_0/r'_0$ . When  $\Lambda = 0$  while substituting  $\kappa'_3 = (16/3)\pi G\rho'_0$  into Eq. (9), it takes the form

$$\frac{v'_0}{r'_0} = \sqrt{\frac{\kappa'_3}{2}} = \sqrt{\frac{8}{3}\pi G\rho'_0}. \quad (10)$$

The expression on the left-hand side of Eq. (9) is independent of the number of mass particles  $N$  that form the sphere. Thus, even if only two particles existed so that  $N = 2$ , the form and behavior of Eq. (9) would be unaltered.

It is important to recognize here that, as stated in the introduction, it is only in the inertial systems that are at rest relative to their local Hubble drift that the cosmological background radiation is observed to be isotropic. The mass particles in Eq. (2a) determine the magnitude of  $H$  and therefore define the motion of these inertial systems and of course other local inertial can have constant velocities relative to them. However, the conservation of energy occurs only within an inertial system. When the first particles are created within a radiation universe, they also define the motion of inertial systems, but only after  $v'_0$  and  $r'_0$  are created. Without an existing inertial system, relative to which mass particle velocities can be defined, the kinetic energy also cannot be defined. Consequently the only energy that could be used by the radiation to create mass particles would be the energy it used to create the rest energy of the masses  $E = mc^2$ . Without a defined inertial system the radiation could not possibly create any energy in addition to rest mass energy which is independent of velocity and the inertial system in which it is defined.

Now consider the physical meaning of Eq.(10). By squaring both sides of Eq. (10) and then multiplying both sides by  $mr'_0{}^2/2$ , while recognizing that  $M = (4/3)\rho'_0\pi r'_0{}^3$  is the total equivalent mass of the radiation inside the spherical shell, it takes the form

$$\frac{1}{2}mv_0'^2 - \frac{GMm}{r'_0} = 0. \quad (11)$$

Eq. (11) states that the total energy of a mass particle due to its kinetic and gravitational potential energies is zero so that all of a mass particle's energy is contained in its rest mass as  $mc^2$ . This is obviously the expression for energy that occurs for a particle that has the minimum velocity required for it to escape from a gravitational mass  $M$  so that  $v'_0 \rightarrow 0$  as  $r'_0 \rightarrow \infty$ . This would hold true for every particle in the particle shell. In this case of mass particles expanding away from the center of the shell which contains the isotropic radiation of density  $\rho'_0$ , the Hubble parameter  $H \rightarrow 0$  as  $r'_0 \rightarrow \infty$ . As a result the Hubble expansion is then balanced as would be expected when  $\Lambda = 0$ .

The initial radius  $r'_0$  of the shell in Eq. (9) at the time  $t' = 0$ , when the relativistic expansion began at the end of the inflationary expansion, depends on the initial velocity of the particles. Assuming  $v'_0 = c/10$  while  $\kappa'_3 = 1.12 \times 10^{54} s^{-2}$ , when the initial density  $\rho'_0 = 10^{60} g/cm^3$  in Eq. (10), yields  $r'_0 = 4.00 \times 10^{-18} cm$  as the required radius that would create this balanced universe.

If in addition to the radial velocities, small tangential velocities had occurred, it can be shown that such velocities could generate a kinematic effect that would yield small values for  $\Lambda$ . The kinematics of both radial and tangential velocities are discussed in a previous paper in [3].

#### IV. Summary

The kinematic definition of the Hubble parameter in Eq.(2a) was shown to be able to take the form  $H = \dot{a}/a$  which is then physically identical to the relativistic definition of the Hubble parameter. Equation (2a) defines the Hubble parameter in terms of the positions and velocities of all mass particles in the universe. The gravitational expression for  $H$  found in the in the standard model of relativity was set equal to the kinematic expression for  $H$  as done in Eq. (9), which occurs after the inflationary expansion. The kinematic definition of the Hubble parameter defines the rate of the universe's expansion, and energy conservation occurs only within an inertial coordinate system. The isotropic form of the cosmic background radiation is observed only within an inertial system that is at rest relative to its local Hubble drift so that  $H$  kinematically defines inertial systems at rest relative to the Hubble drift. When the expansion is about to be generated by the first existing mass particles begins, these particles are forming in a universe with no defined inertial system or systems so that velocity, conservation of energy, or even energy itself cannot be defined relative to an inertial system. The universe's radiation then only creates the rest mass energy of the particles which is independent of both a particle's velocity and its kinetic energy. It requires in turn that the total kinetic and gravitational energy of the particle be zero as shown in Eqs.(10) and (11) where  $\Lambda = 0$ . This condition remains in effect while additional mass particles are subsequently created within their respective local inertial systems that move with the Hubble motion throughout the expanding universe.

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